

# Plošný integrál

**Příklad 1.** Graficky znázorněte plochu  $S$  a vypočtěte:

a)  $\iint_S x \, dS$ , kde  $S = \{[x, y, z] \in \mathbb{R}^3 : x + y + z = 1, x \geq 0, y \geq 0, z \geq 0\}$ ,  $\left[\frac{1}{2\sqrt{3}}\right]$

b)  $\iint_S z^2 \, dS$ , kde  $S = \{[x, y, z] \in \mathbb{R}^3 : z^2 = x^2 + y^2, 0 \leq z \leq 1\}$ ,  $\left[\frac{\pi}{\sqrt{2}}\right]$

c)  $\iint_S z \, dS$ , kde  $S = \{[x, y, z] \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4, x \geq 0, y \geq 0, z \geq 0\}$ ,  $[2\pi]$

d)  $\iint_S y^2 \, dS$ , kde  $S = \{[x, y, z] \in \mathbb{R}^3 : x^2 + z^2 = 4, 0 \leq y \leq 1\}$ ,  $\left[\frac{4\pi}{3}\right]$

e)  $\iint_S xy \, dS$ , kde  $S = \{[x, y, z] \in \mathbb{R}^3 : z = 2x + 3y, 0 \leq x \leq 1, 0 \leq y \leq 1\}$ ,  $\left[\frac{\sqrt{14}}{4}\right]$

f) obsah plochy  $S = \{[x, y, z] \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4, x^2 + y^2 \leq 1, z \geq 0\}$ ,  $[(2 - \sqrt{3}) 4\pi]$

g)  $\iint_S x \, dydz + y \, dx dz + z \, dx dy$ ,  $[2]$

kde  $S = \{[x, y, z] \in \mathbb{R}^3 : 2x + 2y - z = 2, 0 \leq x \leq 1, 0 \leq y \leq 1\}$  a  $\vec{n} = \left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$ ,

h)  $\iint_S x^2 \, dydz + y^2 \, dx dz + z^2 \, dx dy$ ,  $[0]$

kde  $S = \{[x, y, z] \in \mathbb{R}^3 : x^2 + z^2 \leq 9, y = z\}$  a  $\vec{n}([3, 0, 0]) = \left(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ ,

i)  $\iint_S x \, dydz + y \, dx dz + 2z \, dx dy$ ,  $[2\pi]$

kde  $S = \{[x, y, z] \in \mathbb{R}^3 : z = 1 - x^2 - y^2, z \geq 0\}$  a  $\vec{n}([0, 0, 1]) = (0, 0, 1)$ ,

j)  $\iint_S z \, dydz + y \, dx dz - x \, dx dy$ ,  $\left[\frac{16\pi}{3}\right]$

kde  $S = \{[x, y, z] \in \mathbb{R}^3 : y^2 = x^2 + z^2, y \in [0, 2]\}$  a  $\vec{n}([1, 1, 0]) = (-1, 1, 0)$ .