

Cvičení 11

Příklad 1. Vypočtěte (také jako určité integrály)

- $\int 5 \sin x \cos^5 x \, dx \quad \left(\underset{c}{=} \frac{-5 \cos^6 x}{6} \quad \text{pro } x \in \mathbb{R} \right),$
- $\int \frac{\ln^6 x}{x} \, dx \quad \left(\underset{c}{=} \frac{\ln^7 x}{7} \quad \text{pro } x \in (0, \infty) \right),$
- $\int x \sqrt[5]{x^2 + 1} \, dx \quad \left(\underset{c}{=} \frac{5 \sqrt[5]{(x^2 + 1)^6}}{12} \quad \text{pro } x \in \mathbb{R} \right),$
- $\int \frac{\sin^4 x}{\cos^6 x} \, dx \quad \left(\underset{c}{=} \frac{1}{5} \operatorname{tg}^5 x \quad \text{např. pro } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right),$
- $\int \frac{e^{2x}}{e^{2x} + 1} \, dx \quad \left(\underset{c}{=} \frac{\ln(e^{2x} + 1)}{2} \quad \text{pro } x \in \mathbb{R} \right),$
- $\int \frac{\sin x}{\cos x + 2} \, dx \quad \left(\underset{c}{=} -\ln(\cos x + 2) \quad \text{pro } x \in \mathbb{R} \right),$
- $\int \frac{\sin x - \cos x}{\sqrt{\sin x + \cos x}} \, dx$
 $\left(\underset{c}{=} -2\sqrt{\sin x + \cos x} \quad \text{pro } x \in \left(2k\pi - \frac{\pi}{4}, 2k\pi + \frac{3\pi}{4}\right), k \in \mathbb{Z} \right),$
- $\int \frac{5x + 7}{x^2 + 2x + 5} \, dx \quad \left(\underset{c}{=} \frac{5}{2} \ln(x^2 + 2x + 5) + \operatorname{arctg} \frac{x + 1}{2} \quad \text{pro } x \in \mathbb{R} \right),$
- $\int \frac{2x + 1}{(x^2 + 1)^2} \, dx \quad \left(\underset{c}{=} \frac{x - 2}{2x^2 + 2} + \frac{1}{2} \operatorname{arctg} x \quad \text{pro } x \in \mathbb{R} \right),$
- $\int \frac{8x - 31}{x^2 - 9x + 14} \, dx$
 $\left(\underset{c}{=} 3 \ln|x - 2| + 5 \ln|x - 7| \quad \text{v intervalu, který neobsahuje body } x = 2, x = 7 \right),$
- $\int \frac{1}{x^3 + 1} \, dx$
 $\left(\underset{c}{=} \frac{1}{6} \ln \frac{(x + 1)^2}{x^2 - x + 1} + \frac{\sqrt{3}}{3} \operatorname{arctg} \frac{2x - 1}{\sqrt{3}} \quad \text{pro } x \in (-\infty, -1) \text{ nebo } x \in (-1, \infty) \right).$